Sigma-Based Acceptance for "Type 2" Transfer Functions

Igor Volobouev

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I will call the transfer functions normalized over the whole range of observable momenta (as in MTM2.5) "type 1" transfer functions and transfer functions normalized to 1 after all cuts (as in MEAT and MTM3) "type 2" transfer functions.

The procedure we used to calculate the acceptance dependence on JES in units of p_T-dependent sigma is as follows:

1. Smear the partons according to the type 1 transfer functions. This gives us something similar to level 5 jets.

2. For each jet, calculate $\sigma_{JES}(p_{T,jet})$. This is done, basically, by reversing the jet corrections, so that we can use the standard CDF systematics developed for level 0 jets.

3. For a given number of standard deviations, Δ_{JES}, calculate the shifted jet energy scale: $\text{JES} = 1 + \Delta_{JES}\, \sigma_{JES}(p_{T,jet})$.

4. Scale the jets obtained in the first step by 1/JES.

5. See whether every scaled jet in the event passes the $c_0 = 20$ GeV p_T cut. Calculate acceptance as a function of Δ_{JES} using events which pass this cut.

Note that steps 4 and 5 should be equivalent to applying a new cut $c_1 = c_0\,\text{JES}$ to the original event sample with "unscaled" jets. Also, this description is just a rough outline which does not talk about lepton cuts, eta cuts, minimum ΔR between jets, *etc.*

For type 2 transfer functions this procedure is not possible: the smeared jets will automatically end up passing the p_T cut (even though the TFs are implemented with a significant flexibility in choosing the JES and, therefore, the cut). Instead, the jet p_T dependent part of the acceptance should be calculated by finding the average value of the product of the four jet efficiencies. The averaging should be done over $t\bar{t}$ parton-level events generated, for example, by Pythia.

At this time, our jet efficiencies are functions of parton p_T, mass, and JES: $\epsilon = \epsilon_{now}(\mathbf{x}, \mathrm{JES})$, where $\mathbf{x}$ is the vector of parton-level quantities (p_T, mass). Instead, for acceptance calculation purposes, we really want them to be functions of $\mathbf{x}$ and Δ_{JES}: $\epsilon = \epsilon(\mathbf{x}, \Delta_{JES})$. The reason why we can't just substitute JES $= 1 + \Delta_{JES}\, \sigma_{JES}(p_{T,jet})$ is that JES here is a function of $p_{T,jet}$ which is unknown, not p_T of the parton.

If one thinks how to approach the calculation of $\epsilon(\mathbf{x}, \Delta_{JES})$ in case we had type 1 transfer functions available, the procedure would be similar: smear the parton to get the jet, calculate $\sigma_{JES}(p_{T,jet})$, check whether the jet passes the scaled p_t cut c_1. *I believe that this procedure is equivalent to calculating the following integral*:

$$\int W(\mathbf{y}|\mathbf{x})\theta(p_{T,jet} - c_1)d\mathbf{y}$$

where $\mathbf{y}$ stands for jet-level quantities (observed), and $\theta(...)$ is the Heaviside step function. This means

$$\epsilon(\mathbf{x}, \Delta_{JES}) = \int W(\mathbf{y}|\mathbf{x})\theta(p_{T,jet} - c_0(1 + \Delta_{JES}\, \sigma_{JES}(p_{T,jet})))d\mathbf{y}$$

Let's assume for now that integration over angles can be factored out. This is not completely true — typically reconstructed jet angle and p_T are correlated — but simplifies subsequent reasoning a lot. Let's also assume for simplicity that the jet is far away from the $|\eta| = 2$ boundary. Then the angular part of the transfer function integrates to 1, and

$$\epsilon(\mathbf{x}, \Delta_{JES}) = \int W_T(p_{T,jet}|\mathbf{x})\theta(p_{T,jet} - c_0(1 + \Delta_{JES}\, \sigma_{JES}(p_{T,jet})))dp_{T,jet}$$

Note that, at least for small Δ_{JES} values, the equation

$$p_{T,jet} - c_0(1 + \Delta_{JES}\, \sigma_{JES}(p_{T,jet})) = 0$$

has only one solution $p_{T,jet} = p_0(\Delta_{JES})$. This is because the derivative of the function $p_{T,jet} - c_0(1 + \Delta_{JES}\, \sigma_{JES}(p_{T,jet}))$ over $p_{T,jet}$ is $1 - c_0\Delta_{JES}\frac{d\sigma_{JES}(p_{T,jet}))}{dp_{T,jet}}$ which is guaranteed to be positive for small Δ_{JES} magnitudes. Because of this, at least for small Δ_{JES} values the efficiency integral can be rewritten as

$$\epsilon(\mathbf{x}, \Delta_{JES}) = \int_{p_0(\Delta_{JES})}^{\infty} W_T(p_{T,jet}|\mathbf{x})dp_{T,jet}$$

At the same time,

$$\epsilon_{now}(\mathbf{x}, \mathrm{JES}) \equiv \int_{c_0\,\mathrm{JES}}^{\infty} W_T(p_{T,jet}|\mathbf{x})dp_{T,jet}$$

which simply means that $\epsilon(\mathbf{x}, \Delta_{JES}) = \epsilon_{now}(\mathbf{x}, p_0(\Delta_{JES})/c_0)$. Therefore, to calculate $\epsilon(\mathbf{x}, \Delta_{JES})$ we only need to prepare the interpolation table for $p_0(\Delta_{JES})$ values by solving the corresponding equation for various Δ_{JES}, and after that we should be able to use the existing efficiencies.